

Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 9

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

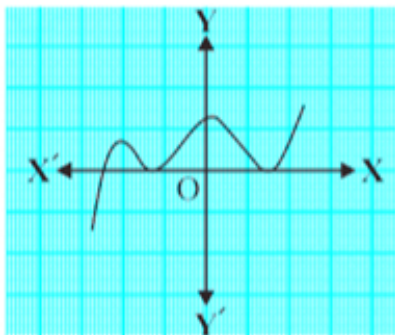
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The number $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is [1]

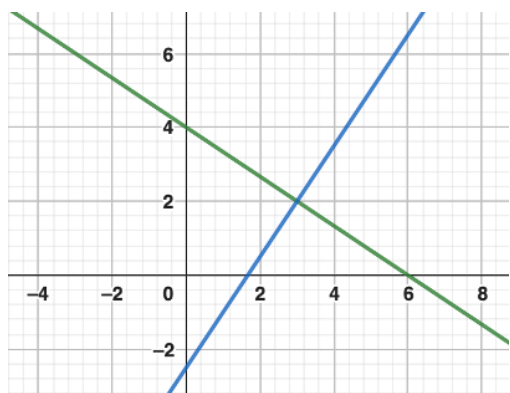
- | | |
|-------------------------|----------------------|
| a) an irrational number | b) an integer |
| c) not a real number | d) a rational number |

2. The graph of $y = p(x)$ in a figure given below, for some polynomial $p(x)$. Find the number of zeroes of $p(x)$. [1]



- | | |
|------|------|
| a) 3 | b) 4 |
| c) 2 | d) 1 |

3. If $2x + 3y = 12$ and $3x - 2y = 5$ then [1]



a) $x = 3, y = 2$

b) $x = 2, y = -3$

c) $x = 2, y = 3$

d) $x = 3, y = -2$

4. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal. Then _____ [1]

a) $2c = a + b$

b) $2a = b + c$

c) $2b = a + c$

d) $\frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

5. The 5th term of an AP is 20 and the sum of its 7th and 11th terms is 64. The common difference of the AP is [1]

a) 4

b) 2

c) 3

d) 5

6. A circle drawn with origin as the centre passes through $(\frac{13}{2}, 0)$. The point which does not lie in the interior of the circle is [1]

a) $(-\frac{3}{4}, 1)$

b) $(2, \frac{7}{3})$

c) $(5, -\frac{1}{2})$

d) $(-6, \frac{5}{2})$

7. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then [1]

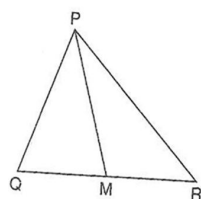
a) $AP = \frac{1}{4}AB$

b) $AP = \frac{1}{2}AB$

c) $AP = \frac{1}{3}AB$

d) $AP = PB$

8. In ΔPQR , if $\frac{PQ}{PR} = \frac{QM}{MR}$, $\angle Q = 75^\circ$ and $\angle R = 45^\circ$. Then the measure of $\angle QPM$ is [1]



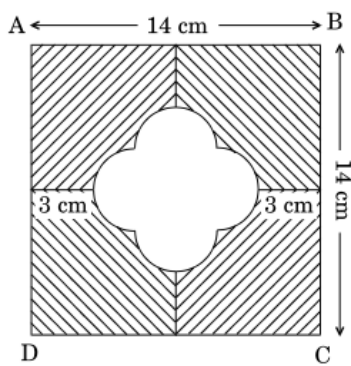
a) 30°

b) 22.5°

c) 45°

d) 60°

9. In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm then the length of AP is [1]



24. Find the value of $\cos 2\theta$, if $2 \sin 2\theta = \sqrt{3}$. [2]

OR

Prove that: $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{2 \sec^2 \theta}{\tan^2 \theta - 1}$

25. From a circular piece of cardboard of radius 3 cm two sectors of 90° have been cut off. Find the perimeter of the remaining portion nearest hundredth centimeters. (Take $\pi = 22/7$). [2]

OR

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Section C

26. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size (in inches) of the tile required that has to be cut and how many such tiles are required? [3]

27. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 2x + 3$, find a polynomial whose roots are [3]

i. $\alpha + 2, \beta + 2$

ii. $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

28. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digits is 3, determine the number. [3]

OR

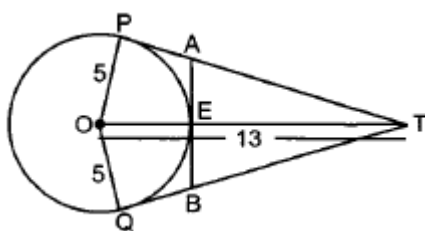
Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

i. intersecting lines

ii. parallel lines

iii. coincident lines

29. In figure, O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB. where TP and TQ are two tangents to the circle. [3]

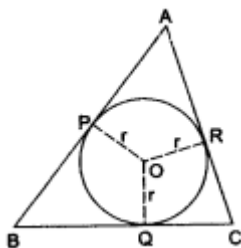


OR

In the given figure, the sides AB, BC and CA of a triangle ABC touch a circle with center O and radius r at P, Q and R respectively. Prove that.

a. $AB + CQ = AC + BQ$

b. $\text{area}(\triangle ABC) = \frac{1}{2}(\text{perimeter of } \triangle ABC) \times r.$



30. Prove that: $\frac{\sin \theta - \cos \theta + 1}{\cos \theta + \sin \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ [3]

31. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained. Find the median height. [3]

Height (in cm)	No. of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Section D

32. Swati can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream. [5]

OR

A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it, having an area of 111 sq m. Find the width of the path.

33. AD and PM are medians of triangles ABC and PQR respectively where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. [5]

34. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/cm^3 and that of the graphite is 2.1 gm/cm^3 . [5]

OR

From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and same radius is hollowed out. Find the total surface area of the remaining solid.

35. Find the mode, median and mean for the following data: [5]

Marks Obtained	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of students	7	31	33	17	11	1

Section E

36. Read the text carefully and answer the questions: [4]

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.



Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

- (i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment?
- (ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment?

OR

If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment?

- (iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment?

37. **Read the text carefully and answer the questions:**

[4]

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electricians's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit



- (i) What is the distance between the grocery store and food cart?
- (ii) What is the distance of the bus stand from the house?

OR

What are the ratio of distances of the house from bus stand to food cart?

- (iii) If the grocery store and electricians shop lie on a line, then what will be the ratio of distance of house from grocery store to that from electrician's shop?

38. **Read the text carefully and answer the questions:**

[4]

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30° coming towards the base of the light house along straight line joining the ship and the base of the

light house. The angle of depression of ship changes to 45° after 6 seconds.



- (i) Find the distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45° .
- (ii) Find the distance between two positions of ship after 6 seconds?

OR

Find the distance of ship from the base of the light house when angle of depression is 30° .

- (iii) Find the speed of the ship?

Solution

Section A

1. (a) an irrational number

Explanation: $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$$= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$
$$= \frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2 - (\sqrt{2})^2}$$
$$= \frac{(\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2}}{5 - 2}$$
$$= \frac{5 + 2 + 2\sqrt{10}}{3}$$
$$= \frac{7 + 2\sqrt{10}}{3}$$

Here $\sqrt{10} = \sqrt{2} \times \sqrt{5}$

Since $\sqrt{2}$ and $\sqrt{5}$ both are an irrational number

Therefore, $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is an irrational number.

2. (a) 3

Explanation: The number of zeroes is 3 as the graph given in the question intersects the x-axis at 3 points.

3. (a) $x = 3, y = 2$

Explanation: We have:

$$2x + 3y = 12 \dots(i)$$

$$3x - 2y = 5 \dots(ii)$$

Now, by multiplying (i) by 2 and (ii) by 3 and then adding them we get:

$$4x + 9x = 24 + 15$$

$$13x = 39$$

$$x = \frac{39}{13} = 3$$

Now putting the value of x in (i), we get

$$2 \times 3 + 3y = 12$$

$$\therefore y = \frac{12-6}{3} = 2$$

- 4.

(b) $2a = b + c$

Explanation: Since the given equation has equal roots.

$$\therefore D = 0$$

$$\Rightarrow (b - c)^2 - 4(c - a)(a - b) = 0$$

$$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$$

$$\Rightarrow (-2a)^2 + (b)^2 + (c)^2 + 2(-2a)(b) + 2bc + 2(-2a)(c) = 0$$

$$\Rightarrow (-2a + b + c)^2 = 0$$

$$\Rightarrow -2a + b + c = 0 \Rightarrow 2a = b + c$$

- 5.

(c) 3

Explanation: $T_5 = 20 \Rightarrow a + 4d = 20 \dots(i)$

$$(T_7 + T_{11}) = 64$$

$$\Rightarrow (a + 6d) + (a + 10d) = 64$$

$$\Rightarrow a + 8d = 32 \dots(ii)$$

Subtracting (i) from (ii), we get

$$4d = 12$$

$$\Rightarrow d = 3$$

6.

(d) $\left(-6, \frac{5}{2}\right)$

Explanation: Distance between $(0, 0)$ and $\left(-6, \frac{5}{2}\right)$

$$\begin{aligned} d &= \sqrt{(-6 - 0)^2 + \left(\frac{5}{2} - 0\right)^2} \\ &= \sqrt{36 + \frac{25}{4}} \\ &= \sqrt{\frac{144+25}{4}} \\ &= \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \end{aligned}$$

So, the point $\left(-6, \frac{5}{2}\right)$ does not lie in the circle.

7.

(b) $AP = \frac{1}{2}AB$

Explanation: $AP = \sqrt{(2 - 4)^2 + (1 - 2)^2}$

$$= \sqrt{4 + 1} = \sqrt{5} = \text{units}$$

$$AB = \sqrt{(8 - 4)^2 + (4 - 2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

Here $AB = 2 \times AP$

$$\therefore AP = \frac{1}{2}AB$$

8. (a) 30°

Explanation: In triangle PQR, $\angle P + \angle Q + \angle R = 180^\circ$

$$\Rightarrow \angle P + 75^\circ + 45^\circ = 180^\circ$$

$$\Rightarrow \angle P = 60^\circ$$

In triangle PQR, if $\frac{PQ}{PR} = \frac{QM}{MR}$, then $\frac{PQ}{PR} = \frac{QM}{MR} = \frac{PM}{PM}$ as it is common.

Therefore, $\triangle PQM \sim \triangle PRM$ (SSS)

Thus $\angle QPM = \angle RPM$

$$\angle P = 60^\circ$$

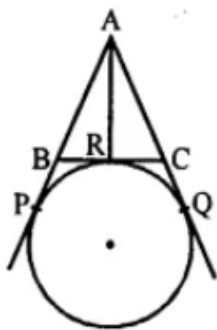
$$\Rightarrow \angle QPM + \angle RPM = 60^\circ$$

$$\Rightarrow 2\angle QPM = 60^\circ$$

$$\Rightarrow \angle QPM = 30^\circ$$

9. (a) 7.5 cm

Explanation: In the given figure, AP, AQ and BC are tangents to the circle.



$$AB = 5 \text{ cm}, AC = 6 \text{ cm}, BC = 4 \text{ cm}$$

Length of AP = ?

BP and BR are the tangents to the circle.

$$BP = BR$$

Similarly, CR and CQ are tangents

$$CR = CQ$$

Similarly, AP and AQ are tangents

$$AP = AQ$$

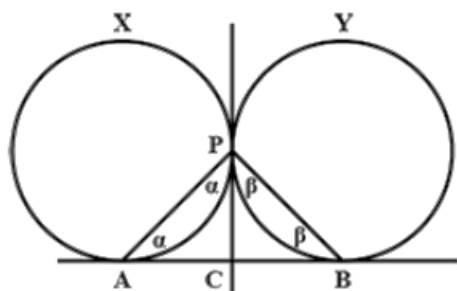
$$AP = AB + BP = AB + BR$$

$$\begin{aligned}
 AQ &= AC + CQ = AC + CR \\
 AP + AQ &= AB + BR + AC + CR = AB + BR + CR + AC \\
 AP + AP &= AB + BC + AC \\
 2AP &= 5 + 4 + 6 = 15 \text{ cm} \\
 AP &= \frac{15}{2} = 7.5 \text{ cm}
 \end{aligned}$$

10.

(c) 90°

Explanation:



Given X and Y are two circles touch each other externally at P. AB is the common tangent to the circles X and Y at point A and B respectively.

Let $\angle CAP = \alpha$ and $\angle CBP = \beta$

$CA = CP$ [lengths of the tangents from an external point C].

In a triangle PAC, $\angle CAP = \angle APC = \alpha$

Similarly, $CB = CP$ and $\angle CPB = \angle PBC = \beta$

Now in the triangle APB,

$\angle PAB + \angle PBA + \angle APB = 180^\circ$ [Sum of the interior angles in a triangle]

$$\alpha + \beta + (\alpha + \beta) = 180^\circ$$

$$2\alpha + 2\beta = 180^\circ$$

$$\alpha + \beta = 90^\circ$$

Therefore, $\angle APB = \alpha + \beta = 90^\circ$

11. (a) $2p$

Explanation: Given: $\sin\theta + \cos\theta = p$

squaring both sides we get

$$\sin^2\theta + \cos^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = p^2$$

$$1 + 2\sin\theta \cos\theta = p^2 (\sin^2\theta + \cos^2\theta = 1)$$

$$2\sin\theta \cos\theta = p^2 - 1 \dots (i)$$

and also $\sec\theta + \operatorname{cosec}\theta = q$ (given)

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta \cos\theta} = q$$

but $\sin\theta + \cos\theta = p \dots$ (given)

$$\frac{p}{\sin\theta \cos\theta} = q \dots (ii)$$

from (i) and (ii) we get

$$q(p^2 - 1) = 2p$$

12. (a) $1 + \frac{z^2}{c^2}$

Explanation: Given: $x = a \sec\theta \cos\phi$, $y = b \sec\theta \sin\phi$

and $z = c \tan\theta$,

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$= \frac{a^2 \sec^2\theta \cos^2\phi}{a^2} + \frac{b^2 \sec^2\theta \sin^2\phi}{b^2}$$

$$= \sec^2\theta (\cos^2\phi + \sin^2\phi) = \sec^2\theta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$= 1 + \tan^2 \theta$$

$$= 1 + \frac{z^2}{c^2}$$

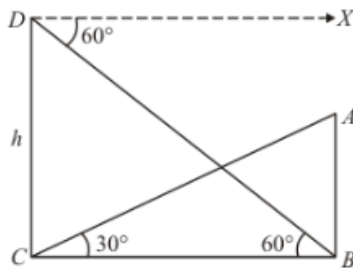
[Given: $z = c \tan \theta$]

13.

(b) $\frac{h}{3}$ m

Explanation: Let AB be the tower and C is a point on the same level as its foot such that $\angle ACB = 30^\circ$

The given situation can be represented as,



Here D is a point h m above the point C.

In $\triangle BCD$,

$$\Rightarrow \tan b = \frac{CD}{CB}$$

$$\Rightarrow \tan 60^\circ = \frac{h}{CB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{CB}$$

$$\Rightarrow CB = \frac{h}{\sqrt{3}}$$

Again in triangle ABC,

$$\tan C = \frac{AB}{CB}$$

$$\Rightarrow \tan 30^\circ = \frac{AB}{\left(\frac{h}{\sqrt{3}}\right)} \text{ [Using (1)]}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\left(\frac{h}{\sqrt{3}}\right)}$$

$$\Rightarrow AB = \frac{h}{3}$$

14. (a) 231 cm^2

Explanation: The angle subtended by the arc = 60°

$$\text{So, area of the sector} = \left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= \left(\frac{441}{6}\right) \times \left(\frac{22}{7}\right) \text{ cm}^2$$

$$= 231 \text{ cm}^2$$

15. (a) $\frac{x}{360} \times \pi r^2$

Explanation: Area of a sector of a circle with radius r and making an angle of x°

$$\text{at the centre} = \frac{x}{360} \times \pi r^2$$

16. (a) 0.00001

Explanation: An event is unlikely to happen. Its probability is very very close to zero but not zero, So it is equal to 0.00001

17.

(d) 4.5

Explanation: Since, Total Probability = 1

$$\therefore \frac{x}{12} + \frac{5}{8} = 1$$

$$\Rightarrow \frac{2x+15}{24} = 1$$

$$\Rightarrow 2x + 15 = 24$$

$$\Rightarrow 2x = 9$$

$$\Rightarrow x = 4.5$$

18.

(c) 0

Explanation: $\sum x_i - \bar{x} = \sum x_i - \sum (\bar{x})$

$\sum (\bar{x}) = n (\bar{x})$ by definition

But $\bar{x} = \frac{(\sum x_i)}{n}$ by definition

$$\text{So, } \sum x_i - \sum (\bar{x}) = \sum x_i - n \frac{(\sum x_i)}{n}$$

$$\text{which is equal to } = (\sum x_i) - (\sum x_i) = 0$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation: Assertion: Even natural numbers divisible by 5 are 10, 20, 30, 40, ...

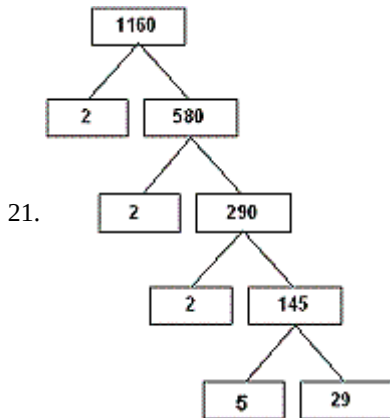
They form an A.P. with,

$$a = 10, d = 10$$

$$S_{100} = \frac{100}{2} [2(10) + 99(10)] = 50500$$

So, reason is correct.

Section B



22. According to question,

$$AB = 10.8 \text{ cm, } AD = 6.3 \text{ cm, } AC = 9.6 \text{ cm, } EC = 4 \text{ cm}$$

$$\text{Now } \frac{AD}{DB} = \frac{AD}{AB-AD} = \frac{6.3}{10.8-6.3} = 1.4$$

$$\text{And } \frac{AE}{EC} = \frac{AC-EC}{EC} = \frac{9.6-4}{4} = 1.4$$

$$\text{Therefore, } \frac{AD}{DB} = \frac{AE}{EC}$$

Hence by the converse of Thales theorem, $DE \parallel BC$

23. Given ABCD is a square of side 14 cm each

and four semicircles of same radii

To find : area of shaded region proof

$$\text{Area of semicircle with radius 2 cm} = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times 3.14 \times (2)^2$$

$$= \frac{1}{2} \times 3.14 \times 4$$

$$= 3.14 \times 2 \text{ cm}^2$$

In figure, there are 4 semi-circles of same radius 2 cm

$$\text{So, Area of 4 Semi-circle} = 4 \times 3.14 \times 2 \text{ cm}^2$$

$$= 8 \times 3.14$$

$$= 25.12 \text{ cm}^2$$

$$\text{Area of smaller square PQRS} = (\text{side})^2$$

$$= (4)^2$$

$$= 16$$

$$\text{Area of shaded region} = \text{Area of square ABCD} - \text{Area of square PQRS} - \text{Area of 4 semi-circles}$$

$$= 196 - 16 - 25.12$$

$$= 154.88 \text{ cm}^2$$

24. Given, $2\sin 2\theta = \sqrt{3}$

$$\text{or, } \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \sin 60^\circ$$

$$2\theta = 60^\circ$$

Hence, $\cos 2\theta = \cos 60^\circ = \frac{1}{2}$

OR

LHS

$$\begin{aligned} &= \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2}{(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)} \\ &= \frac{(\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta) + (\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta)}{\sin^2 \theta - \cos^2 \theta} \\ &= \frac{2(\sin^2 \theta + \cos^2 \theta)}{\sin^2 \theta - \cos^2 \theta} = \frac{2}{\sin^2 \theta - \cos^2 \theta} \because \sin^2 \theta + \cos^2 \theta = 1 \\ &= \frac{\frac{2}{\cos^2 \theta}}{\frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta}} \text{ Dividing the numerator and denominator by } \cos^2 \theta \\ &= \frac{2 \sec^2 \theta}{\tan^2 \theta - 1} = RHS \end{aligned}$$

25. Radius of the circular piece of cardboard(r) = 3 cm

∴ Two sectors of 90° each have been cut off

∴ We get a semicircular cardboard piece

∴ Perimeter of arc ACB

$$\begin{aligned} &= \frac{1}{2}(2\pi r) = \pi r \\ &= \frac{22}{7} \times 3 = \frac{66}{7} = 9.428 \text{ cm} \end{aligned}$$

OR

Here, r = 14 cm and $\theta = \frac{90^\circ}{3} = 30^\circ$

$$\begin{aligned} \therefore \text{Area swept} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \\ &= \frac{154}{3} \text{ cm}^2 \end{aligned}$$

Section C

26. **Given:** Size of bathroom = 10 ft by 8 ft.

= (10 × 12) inch by (8 × 12) inch

= 120 inch by 96 inch

Area of bathroom = 120 inch by 96 inch

To find the largest size of tile required, we find HCF of 120 and 96.

By applying Euclid’s division lemma

120 = 96 × 1 + 24

96 = 24 × 4 + 0

Therefore, HCF = 24

Therefore, Largest size of tile required = 24 inches

no. of tiles required = $\frac{\text{area of bathroom}}{\text{area of a tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20$ tiles

Hence number of tiles required is 20 and size of tiles is 24 inches.

27. Given polynomial is

$f(x) = x^2 - 2x + 3$

Compare with $ax^2 + bx + c$, we get

a = 1, b = -2 and c = 3

Sum of the zeroes = $\alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$

Product of the zeroes = $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

i. Sum of the zeroes of new polynomial = $(\alpha + 2) + (\beta + 2)$

= $\alpha + \beta + 4$

= $2 + 4 = 6$

Product of the zeroes of new polynomial = $(\alpha + 2)(\beta + 2)$

= $\alpha\beta + 2\alpha + 2\beta + 4$

= $\alpha\beta + 2(\alpha + \beta) + 4$

= $3 + 2(2) + 4$

= 11

So, quadratic polynomial is: $x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$

$$= x^2 - 6x + 11$$

Hence, the required quadratic polynomial is $f(x) = (x^2 - 6x + 11)$

ii. Sum of the zeroes of new polynomial = $\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$$\begin{aligned} &= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)} \\ &= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta + \beta - \alpha - 1}{(\alpha+1)(\beta+1)} \\ &= \frac{\alpha\beta - 1 + \alpha\beta - 1}{\alpha\beta + \alpha + \beta + 1} \\ &= \frac{3 - 1 + 3 - 1}{3 + 1 + 2} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

Product of the zeroes of new polynomial = $\frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1}$

$$\begin{aligned} &= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)} \\ &= \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1} \\ &= \frac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1} \\ &= \frac{3 - 2 + 1}{3 + 2 + 1} \\ &= \frac{2}{6} = \frac{1}{3} \end{aligned}$$

So, the quadratic polynomial is, $x^2 - (\text{sum of the zeroes})x + (\text{product of the zeroes})$
 $= x^2 - \frac{2}{3}x + \frac{1}{3}$

Thus, the required quadratic polynomial is $f(x) = k \left(x^2 - \frac{2}{3}x + \frac{1}{3} \right)$.

28. Let the ten's and unit digit be y and x respectively.

So the number is $10y + x$

The number when digits are reversed becomes $10x + y$

So, $7(10x + y) = 4(10y + x)$

or, $70x + 7y = 40y + 4x$

or, $70x - 4x = 40y - 7y$

or $66x = 33y$

$\Rightarrow 2x = y \dots\dots(i)$

The difference of the digits is 3

$y - x = 3$

$2x - x = 3$

$x = 3$

$x = 3$ and $y = 6$

Hence the number is 63

OR

Given, linear equation is $2x + 3y - 8 = 0 \dots(i)$

Given: $2x + 3y - 8 = 0 \dots (i)$

i. For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

\therefore Any line intersecting with eq (i) may be taken as $3x + 2y - 9 = 0$
or $3x + 2y - 7 = 0$

ii. For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

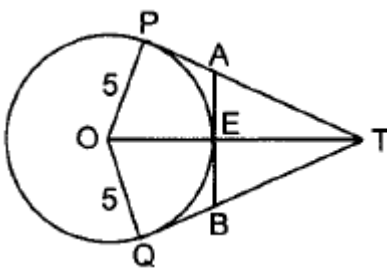
\therefore Any line parallel with eq(i) may be taken as $6x + 9y + 7 = 0$
or $2x + 3y - 2 = 0$

iii. For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

\therefore Any line coincident with eq (i) may be taken as $4x + 6y - 16 = 0$
or $6x + 9y - 24 = 0$

29. According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E.



$\therefore OP \perp TP$ [Radius from point of contact of the tangent]

$\therefore \angle OPT = 90^\circ$

In right $\triangle OPT$ *

$$OT^2 = OP^2 + PT^2$$

$$\Rightarrow (13)^2 = (5)^2 + PT^2 \Rightarrow PT = 12 \text{ cm}$$

Let $AP = x$ cm $AE = AP \Rightarrow AE = x$ cm

and $AT = (12 - x)$ cm

$$TE = OT - OE = 13 - 5 = 8 \text{ cm}$$

$\therefore OE \perp AB$ [Radius from the point of contact]

$\therefore \angle AEO = 90^\circ \Rightarrow \angle AET = 90^\circ$

In right $\triangle AET$,

$$AT^2 = AE^2 + ET^2$$

$$(12 - x)^2 = x^2 + 8^2$$

$$\Rightarrow 144 + x^2 - 24x = x^2 + 64$$

$$\Rightarrow 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3} \text{ cm}$$

Also $BE = AE = \frac{10}{3}$ cm

$$\Rightarrow AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3} \text{ cm}$$

OR

We know that the lengths of tangents from an exterior point to a circle are equal.

$AP = AR$, ... (i) [tangents from A]

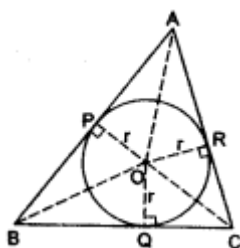
$BP = BQ$, ... (ii) [tangents from B]

$CQ = CR$, ... (iii) [tangents from C]

- a. $AB + CQ = AP + BP + CQ$
 $= AR + BQ + CR$ [using (i), (ii) and (iii)]
 $= (AR + CR) + BQ = AC + BQ$.

b. Join OA, OB and OC.

$$\begin{aligned} \text{Area } (\triangle ABC) &= \text{area } (\triangle OAB) \\ &+ \text{area } (\triangle OBC) \\ &+ \text{area } (\triangle OCA) \\ &= \left(\frac{1}{2} \times AB \times OP\right) \\ &+ \left(\frac{1}{2} \times BC \times OQ\right) \\ &+ \left(\frac{1}{2} \times CA \times OR\right) \\ &= \left(\frac{1}{2} \times AB \times r\right) + \left(\frac{1}{2} \times BC \times r\right) + \left(\frac{1}{2} \times CA \times r\right) \\ &= \frac{1}{2}(AB + BC + CA) \times r \\ &= \frac{1}{2}(AB + BC + CA) \times r \\ &= \frac{1}{2}(\text{perimeter of } \triangle ABC) \times r \end{aligned}$$



30. We have to prove that, $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$

$$\begin{aligned} \text{Taking L.H.S.} &= \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} \\ &= \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} \quad [\text{dividing the numerator and denominator by } \cos \theta] \\ &= \frac{(\tan \theta + \sec \theta) - 1}{(\tan \theta - \sec \theta) + 1} \\ &= \frac{\{(\tan \theta + \sec \theta) - 1\}(\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \quad [\text{Multiplying and dividing by } (\tan \theta - \sec \theta)] \\ &= \frac{(\tan^2 \theta - \sec^2 \theta) - (\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \quad [\because (a - b)(a + b) = a^2 - b^2] \\ &= \frac{-1 - \tan \theta + \sec \theta}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \quad [\because \tan^2 \theta - \sec^2 \theta = -1] \\ &= \frac{-(\tan \theta - \sec \theta + 1)}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \\ &= \frac{-1}{\tan \theta - \sec \theta} \\ &= \frac{1}{\sec \theta - \tan \theta} \\ &= \text{R.H.S.} \end{aligned}$$

Hence Proved.

31. We have,

Class Intervals	Frequency (f)	C.F
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51
	$N = \sum f = 51$	

Here, $\frac{N}{2} = \frac{51}{2} = 25.5$ which is in the class 145-150

Here, $l_1 = 145$, $h = 5$, $N = 51$, $C = 11$, $F = 18$

$$\therefore \text{Median} = l_1 + \frac{\frac{N}{2} - C}{f} \times h$$

$$= 145 + \frac{25.5 - 11}{18} \times 5$$

$$= 145 + \frac{72.5}{18} \Rightarrow 149.03$$

\therefore Median height of the girls = 149.03

Section D

32. Let the speed of the stream be x km/hr.

Speed of boat upstream = $(5 - x)$ km/hr.

Speed of boat downstream = $(5 + x)$ km/hr.

Time taken to go upstream = $\frac{5.25}{5-x}$ hours.

Time taken to go downstream = $\frac{5.25}{5+x}$ hours.

According to question,

$$\therefore \frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$$

$$\Rightarrow 5.25 \left[\frac{1}{5-x} - \frac{1}{5+x} \right] = 1$$

$$\Rightarrow \frac{21}{4} \left[\frac{5+x-5-x}{(5-x)(5+x)} \right] = 1$$

$$\Rightarrow \frac{21}{4} \times \frac{2x}{25-x^2} = 1$$

$$\Rightarrow 21x = 50 - 2x^2$$

$$\Rightarrow 2x^2 + 21x - 50 = 0$$

$$\Rightarrow 2x^2 + 25x - 4x - 50 = 0$$

$$\Rightarrow x(2x + 25) - 2(2x + 25) = 0$$

$$\Rightarrow (2x + 25)(x - 2) = 0$$

$$\Rightarrow x - 2 = 0, 2x + 25 = 0$$

$$\Rightarrow x = 2 \left[\because x \neq -\frac{25}{2} \text{ as } x > 0 \right]$$

Hence, the speed of the stream is 2 km/hr.

OR

Let the width of the path be x m

Length of the field including the path = $(20 + 2x)$ m

Breadth of the field including the path = $(14 + 2x)$ m.

Area of rectangle = $L \times B$

Area of the field including the path = $(20 + 2x)(14 + 2x) \text{ m}^2$.

Area of the field excluding the path = $(20 \times 14) \text{ m}^2 = 280 \text{ m}^2$.

\therefore Area of the path = $(20 + 2x)(14 + 2x) - 280$

$$(20 + 2x)(14 + 2x) - 280 = 111$$

$$\Rightarrow 4x^2 + 68x - 111 = 0$$

Factorise the equation,

$$\Rightarrow 4x^2 + 74x - 6x - 111 = 0$$

$$\Rightarrow 2x(2x + 37) - 3(2x + 37) = 0$$

$$\Rightarrow (2x + 37)(2x - 3) = 0$$

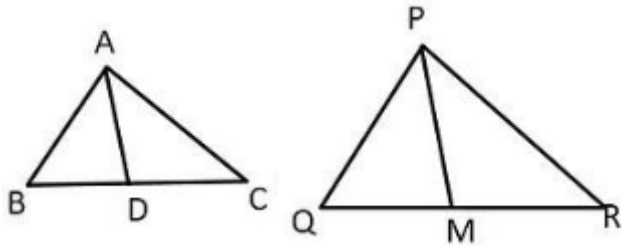
$$\Rightarrow x = -\frac{37}{2} \text{ or } x = \frac{3}{2}$$

As width can't be negative.

$$\Rightarrow x = \frac{3}{2} = 1.5$$

Therefore, the width of the path is 1.5 m.

33.



Given: In $\triangle ABC$ and $\triangle PQR$, AD is the median of $\triangle ABC$, PM is the median of $\triangle PQR$ and $\triangle ABC \sim \triangle PQR$.

To Prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof:

Since AD is the median

$$BD = CD = \frac{1}{2}BC$$

Similarly, PM is the median

$$QM = RM = \frac{1}{2}QR$$

Now,

$\triangle ABC \sim \triangle PQR$. (\because given)

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad (\because \text{Corresponding sides of similar triangle are proportional})$$

So,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{2BD}{2QM} \quad (\text{Since } AD \text{ \& } PM \text{ are medians})$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots\dots\dots(1)$$

Also, since $\triangle ABC \sim \triangle PQR$.

$$\angle B = \angle Q \quad (\because \text{Corresponding angles of similar triangles are equal})\dots\dots\dots(2)$$

Now,

In $\triangle ABD$ & $\triangle PQM$

$$\angle B = \angle Q \quad [\because \text{from (2)}]$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad [\because \text{from (1)}]$$

Hence by SAS similarly,

$$\triangle ABD \sim \triangle PQM$$

Since corresponding sides of similar triangles are proportional,

$$\frac{AB}{PQ} = \frac{AD}{PM}$$

Hence proved.

34. We have, Diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$ cm

$$\therefore \text{Radius of graphite (r)} = \frac{1}{20} \text{ cm} = 0.05 \text{ cm}$$

Length of the graphite cylinder = 10 cm

$$\text{Volume of the graphite cylinder} = \frac{22}{7} \times (0.05)^2 \times 10$$

$$= 0.0785 \text{ cm}^3$$

Weight of graphite = Volume \times Specific gravity

$$= 0.0785 \times 2.1$$

$$= 0.164 \text{ gm}$$

$$\text{Diameter of pencil} = 7 \text{ mm} = \frac{7}{10} \text{ cm} = 0.7 \text{ cm}$$

$$\therefore \text{Radius of pencil} = \frac{7}{20} \text{ cm} = 0.35 \text{ cm}$$

and, Length of pencil = 10 cm

$$\therefore \text{Volume of pencil} = \pi r^2 h$$

$$= \frac{22}{7} \times (0.35)^2 \times 10 \text{ cm}^3 = 3.85 \text{ cm}^3$$

Volume of wood = volume of the pencil - volume of graphite

$$= (3.85 - 0.164) \text{ cm}^3 = 3.686 \text{ gm}$$

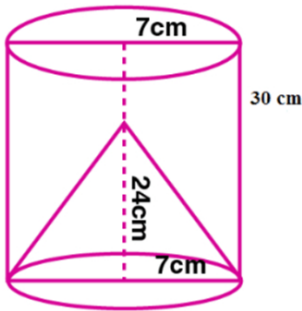
\therefore Weight of wood = volume density

$$= 3.686 \times 0.7 = 3.73$$

Hence, Total weight = (3.73 + 0.164) gm = 3.894 gm.

OR

As the conical cavity is drilled out from the cylinder, so the volume of the remaining solid can be calculated by subtracting the volume of cylinder and cone



Given height of the cylinder, $H = 30$ cm

Radius of the cylinder, $r = 7$ cm

Height of cone, $h = 24$ cm

Radius of cone, $r = 7$ cm

Slant height of the cone, $l = \sqrt{(h^2 + r^2)}$

$$l = \sqrt{(24^2 + 7^2)}$$

$$l = \sqrt{(576 + 49)}$$

$$l = \sqrt{(625)}$$

$$l = 25 \text{ cm}$$

Volume of the remaining solid = Volume of the cylinder - Volume of the cone

$$= \pi r^2 H - \left(\frac{1}{3}\right) \pi r^2 h$$

$$= \pi r^2 \left(H - \frac{h}{3}\right)$$

$$= \left(\frac{22}{7}\right) \times 72 \times \left(30 - \frac{24}{3}\right)$$

$$= (22 \times 7) \times (30 - 8)$$

$$= (154) \times (22)$$

$$= 3388 \text{ cm}^3$$

Volume of the remaining solid is 3388 cm^3 .

Total surface area of the remaining solid = Curved surface area of cylinder + surface area of top of the cylinder + curved surface area of the cone

Total surface area of the remaining solid

$$\begin{aligned}
 &= 2\pi rH + \pi r^2 + \pi rl \\
 &= \pi r(2H + r + l) \\
 &= \left(\frac{22}{7}\right) \times 7(2 \times 30 + 7 + 25) \\
 &= 22 \times (60 + 32) \\
 &= 22 \times 92 \\
 &= 2024 \text{ cm}^2
 \end{aligned}$$

Hence the total surface area of the remaining solid is 2024 cm².

35. Table:

Class	Frequency	Mid value x_i	$f_i x_i$	Cumulative frequency
25 - 35	7	30	210	7
35 - 45	31	40	1240	38
45 - 55	33	50	1650	71
55 - 65	17	60	1020	88
65 - 75	11	70	770	99
75 - 85	1	80	80	100
	N = 100		$\sum f_i x_i = 4970$	

i. Mean

$$\frac{\sum f_i x_i}{\sum f_i} = \frac{4970}{100} = 49.70$$

ii. $N = 100$, $\frac{N}{2} = 50$

Median Class is 45 - 55

$$l = 45, h = 10, N = 100, c = 38, f = 33$$

$$\therefore \text{Median} = l + h \left(\frac{\frac{N}{2} - c}{f} \right)$$

$$= 45 + \left\{ 10 \times \frac{50 - 38}{33} \right\}$$

$$= 45 + 3.64 = 48.64$$

iii. we know that, Mode = 3 × median - 2 × mean

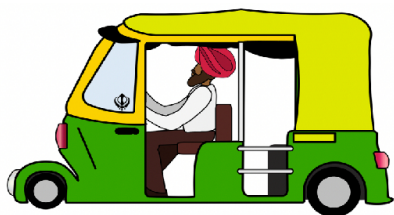
$$= 3 \times 48.64 - 2 \times 49.70$$

$$= 145.92 - 99.4 = 46.52$$

Section E

36. Read the text carefully and answer the questions:

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.



Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 which forms an AP, with first term, $a = 1000$ and common difference, $d = 1100 - 1000 = 100$

Now, amount paid in the 30th installment,

$$a_{30} = 1000 + (30 - 1)100 = 3900 \{a_n = a + (n - 1)d\}$$

- (ii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = ₹ 1300 which forms an AP, with first term, $a = 1000$ and common difference, $d = 1100 - 1000 = 100$

Amount paid in 30 instalments,

$$S_{30} = \frac{30}{2} [2 \times 1000 + (30 - 1)100] = 73500$$

Hence, remaining amount of loan that he has to pay = $118000 - 73500 = 44500$ Rupees

OR

Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = ₹ 1300 which forms an AP, with first term, $a = 1000$ and common difference, $d = 1100 - 1000 = 100$

If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment

Then $a = 1000$, $d = 200$ and $n = 40$

$$a_{40} = a + (n - 1)d$$

$$\Rightarrow a_{40} = 1000 + (40 - 1)200$$

$$\Rightarrow a_{40} = 880$$

- (iii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = ₹ 1300 which forms an AP, with first term, $a = 1000$ and common difference, $d = 1100 - 1000 = 100$

Amount paid in 100 instalments

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{100}{2} [2 \times 1000 + (100 - 1)100]$$

$$\Rightarrow S_n = 100000 + 9900$$

$$\Rightarrow 109900$$

37. Read the text carefully and answer the questions:

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electrician's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit



- (i) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, grocery store is at (2, 3) and food cart is at (6, -8)

$$\therefore \text{Required distance} = \sqrt{(6 - 2)^2 + (-8 - 3)^2}$$

$$= \sqrt{4^2 + 11^2} = \sqrt{16 + 121} = \sqrt{137} \text{ cm}$$

- (ii) Consider the house is at origin (0, 0), then coordinates of the grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Required distance

$$= \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$$

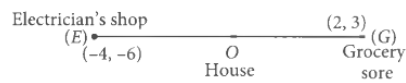
OR

Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Since, (0, 0) is the mid-point of (-6, 8) and (6, -8), therefore both bus stand and food cart are at equal distances from the house. Hence, required ratio is 1 : 1.

(iii) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, -8) and (-6, 8).

Let O divides EG in the ratio k : 1, then



$$0 = \frac{2k-4}{k+1}$$

$$\Rightarrow 2k = 4$$

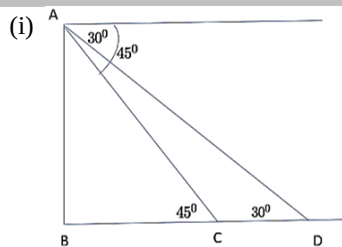
$$\Rightarrow k = 2$$

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE i.e., 1 : 2.

38. Read the text carefully and answer the questions:

An observer on the top of a 40m tall light house (including height of the observer) observes a ship at an angle of depression 30° coming towards the base of the light house along straight line joining the ship and the base of the light house. The angle of depression of ship changes to 45° after 6 seconds.



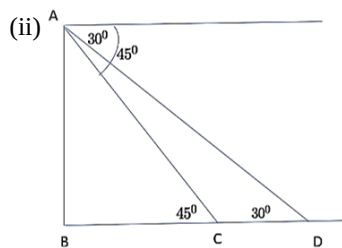
The distance of ship from the base of the light house after 6 seconds from the initial position when angle of depression is 45° .

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{40}{BC}$$

$$\Rightarrow BC = 40 \text{ m}$$



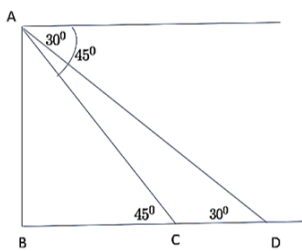
The distance between two positions of ship after 6 seconds

$$CD = BD - BC$$

$$\Rightarrow CD = 40\sqrt{3} - 40 = 40(\sqrt{3} - 1)$$

$$\Rightarrow CD = 29.28 \text{ m}$$

OR



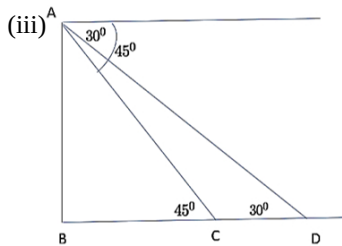
The distance of ship from the base of the light house when angle of depression is 30° .

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{BD}$$

$$\Rightarrow BD = 40\sqrt{3} \text{ m}$$



$$\text{Speed of ship} = \frac{\text{Distance}}{\text{Time}} = \frac{29.28}{6} = 4.88 \text{ m/sec}$$